Numerical Methods

Numerical Integration:

1. Trapezium rule: Form trapeziums along the curve.

$$F(x)=\frac{h}{2}\cdot[y_0+2\cdot y_1+2\cdot y_2+\cdots+2\cdot y_{n-1}+y_n]$$

2. Simpson's one-third rule: *(n* must be even)

$$F(x)=\frac{h}{3}\cdot[y_0+4\cdot y_1+2\cdot y_2+\cdots+4\cdot y_{n-1}+y_n]$$

3. Simpson's three-eighth rule:

$$F(x)=\frac{3\mathrm{h}}{8}\cdot[y_0+y_n+3\cdot(y_1+y_2+y_4+y_5+y_7+y_8+\cdots)+2\cdot(y_3+y_6+y_9+\cdots)]$$

4. Weddle's rule:

$$F(x)=\frac{3\mathrm{h}}{10}\cdot[y_1+5\mathrm{y}_2+y_3+6\mathrm{y}_4+y_5+5\mathrm{y}_6+y_7]$$

ODEs:

5. Milne's Method:

$$y_{n+1}=y_{n-3}+\frac{4\mathrm{h}}{3}\cdot(2\mathrm{y}'_n-y'_{n-1}+2\mathrm{y}'_{n-2})$$
$$y_{n+1}=y_{n-1}+\frac{h}{3}\cdot(y'_{n-1}+4\mathrm{y}'_n+y'_{n+1})$$

6. Adam-Bashforth Method:

$$y_{n+1}=y_n+\frac{h}{24}\cdot[55\mathrm{y}'_n-59\mathrm{y}'_{n-1}+37\mathrm{y}'_{n-2}-9\mathrm{y}'_{n-3}]$$
$$y_{n+1}=y_n+\frac{h}{24}\cdot[9\mathrm{y}'_{n+1}+19\mathrm{y}'_n-5\mathrm{y}'_{n-1}+y'_{n-2}]$$

7. Euler's Method:
$$y_{n+1}=y_n+h\cdot f(x_n,y_n)$$

8. Eüler's Modified Method:
$$y_{n+1}=y_n+h\cdot f(x_n,y_n)$$
$$y_{n+1}=y_n+\frac{h}{2}\cdot[f(x_{n+1},y_{n+1})+f(x_n,y_n)]$$

9. Runge-Kutta Method:

$$k_1 = h \cdot f(x_n, y_n)$$

$$k_2 = h \cdot f(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$$

$$k_4 = h \cdot f(x_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + \frac{1}{6} \cdot (k_1 + 2k_2 + 2k_3 + k_4)$$

10. Midpoint Method:

$$k_1 = h \cdot f(x_n, y_n)$$

$$k_2 = h \cdot f(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$$

$$y_{n+1} = y_n + k_2$$

Algebraic Equations:

11. Regula – Falsi:

Find two point a, b such that the sign of the function is opposite at the two points. Then,

$$x_1 = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$$

Then, according as $f(x_1)$ has opposite sign with $f(a)$ or $f(b)$ repeat the process with a or b and x_1 until the error is acceptable.

12. Bisection:

Find two point a, b such that the sign of the function is opposite at the two points. Then,

$x_1 = \frac{a+b}{2}$ and according as $f(x_1)$ has opposite sign with $f(a)$ or $f(b)$, choose a or b to repeat the process with x_1

13. Newton – Raphson:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

14. Secant:

$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \cdot f(x_n)$ Approximating the derivative from previous method

15. Graeffe's root squaring:
Separate terms of odd and even power on either side of the equation. Square both sides and replace x^2 by y. Now separate terms of odd and even power. Square both sides and replace y^2 by z. Now write the equation as:
$z^n + a_1 z^{n-1} + a_2 z^{n-1} + \cdots + a_{n-1} z + a_n = 0$. Then the roots are $\sqrt[4]{-a_1}$, $\sqrt[4]{\frac{-a_2}{a_1}}$, and in general $\sqrt[4]{\frac{-a_k}{a_{k-1}}}$.

16. Müller's Method:
x_k, x_{k-1}, x_{k-2} are 3 distinct approximations to a root. y_k, y_{k-1}, y_{k-2} are the corresponding values of y.

$$\lambda = \frac{(x - x_k)}{(x_k - x_{k-1})}, \quad \lambda_k = \frac{(x_k - x_{k-1})}{(x_{k-1} - x_{k-2})}, \quad \mu_k = \lambda_k + 1$$

$$\phi_k = y_{k-2} \cdot \lambda_k^2 - y_{k-1} \cdot \mu_k^2 + y_k \cdot (\lambda_k + \mu_k)$$

$$\lambda = \frac{-2 y_k \mu_k}{\phi \pm \sqrt{(\phi - 4 y_k \mu_k \lambda_k \cdot (y_{k-2} \lambda_k - y_{k-1} \mu_k + y_k))}}$$

$$x = x_k + \lambda \cdot (x_k - x_{k-1})$$

Interpolation:

16. Lagrange's Interpolating polynomial:
$$p(x) = \sum y_i \cdot \prod \frac{(x - x_k)}{(x_i - x_k)}$$

17. Newton's Interpolating polynomial: (using divided differences)

$$f(x) = y_0 + \frac{\Delta y_0}{h \cdot 1!} \cdot (x - x_0) + \frac{\Delta^2 y_0}{h^2 \cdot 2!} \cdot (x - x_0)(x - x_1) + \frac{\Delta^3 y_0}{h^3 \cdot 3!} \cdot (x - x_0)(x - x_1)(x - x_2) + \cdots$$

if the differences are constant in x.

$f(x) = y_0 + b_0 (x - x_0) + b_1 (x - x_0)(x - x_1) + \cdots$, where

$$b_0 = \frac{f(x_1) - f(x_0)}{x - x_0}$$

$$b_1 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$ and so on.